ASSESSING THE VALIDITY OF THE ROTATING CYLINDER ANALOGY TO PIPE FLOW FOR DETERMINING THE EFFECT OF FLOW VELOCITY ON CORROSION RATES

Jason Howison, Department of Mechanical Engineering, The Citadel, Charleston, SC
Elissa Trueman, Naval Surface Warfare Center, Carderock Division, West Bethesda, MD

ABSTRACT
Corrosion in pipes is often studied experimentally via external flows about rotating cylinder electrodes (RCEs). Due to limited physical access inside of pipes, an external configuration allows for better placement of sensors and data acquisition so long as the hydrodynamics about the RCE are representative of the hydrodynamics inside of the pipe. The objective of this research is to determine how well this analogy really is in studying corrosion related phenomena in pipes. The Navier-Stokes equations are solved analytically and computationally for flows about a rotating cylinder and inside of a pipe. A linear relationship is found to exist between the cylinder angular velocity and the average pipe flow velocity when the wall shear stresses are matched. These results will be used to help direct an experimental effort to investigate the rotating cylinder analogy to pipe flow. The authors hope that this work will lead to a better understanding of corrosion phenomena that will eventually lead to better procedures for corrosion prevention and treatment.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$</td>
<td>concentration</td>
</tr>
<tr>
<td>$D$</td>
<td>diameter, diffusion coefficient</td>
</tr>
<tr>
<td>$e$</td>
<td>absolute charge of electron</td>
</tr>
<tr>
<td>$f$</td>
<td>friction factor</td>
</tr>
<tr>
<td>$F$</td>
<td>body force vector</td>
</tr>
<tr>
<td>$g$</td>
<td>acceleration due to gravity</td>
</tr>
<tr>
<td>$J$</td>
<td>species flux</td>
</tr>
<tr>
<td>$k$</td>
<td>turbulent kinetic energy, Boltzmann’s constant</td>
</tr>
<tr>
<td>$L$</td>
<td>pipe length</td>
</tr>
<tr>
<td>$n$</td>
<td>number of cylinder radii</td>
</tr>
<tr>
<td>$p$</td>
<td>pressure</td>
</tr>
<tr>
<td>$r$</td>
<td>radial direction</td>
</tr>
<tr>
<td>$R$</td>
<td>cylinder radius, reaction rate</td>
</tr>
<tr>
<td>$Re_D$</td>
<td>Reynolds number for pipe flow</td>
</tr>
<tr>
<td>$Re_r$</td>
<td>Reynolds number for cylinder flow</td>
</tr>
<tr>
<td>$t$</td>
<td>time</td>
</tr>
<tr>
<td>$T$</td>
<td>temperature</td>
</tr>
<tr>
<td>$u$</td>
<td>velocity vector</td>
</tr>
<tr>
<td>$V$</td>
<td>average velocity in pipe flow</td>
</tr>
<tr>
<td>$v_z$</td>
<td>velocity component in axial direction</td>
</tr>
<tr>
<td>$v_\theta$</td>
<td>velocity component in circumferential direction</td>
</tr>
<tr>
<td>$z$</td>
<td>axial direction in pipe flow, ion valence</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>activity coefficient</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>pipe roughness</td>
</tr>
<tr>
<td>$\epsilon_0$</td>
<td>permittivity of a vacuum</td>
</tr>
<tr>
<td>$\epsilon_r$</td>
<td>fluid dielectric constant</td>
</tr>
<tr>
<td>$\theta$</td>
<td>circumferential direction</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>radioactive decay constant</td>
</tr>
<tr>
<td>$\mu$</td>
<td>dynamic viscosity</td>
</tr>
<tr>
<td>$\nu$</td>
<td>kinematic viscosity</td>
</tr>
<tr>
<td>$\rho$</td>
<td>density</td>
</tr>
<tr>
<td>$\rho_E$</td>
<td>charge density</td>
</tr>
<tr>
<td>$\tau_{cyt}$</td>
<td>shear stress on cylinder wall</td>
</tr>
<tr>
<td>$\tau_{pipe}$</td>
<td>shear stress on pipe wall</td>
</tr>
<tr>
<td>$\Psi$</td>
<td>electrical potential</td>
</tr>
<tr>
<td>$\omega$</td>
<td>turbulent dissipation rate, angular velocity</td>
</tr>
</tbody>
</table>
INTRODUCTION
Corrosion in pipes is a problem for the oil and gas industry [1,2], marine vessels [3,4], and many other applications. The difficulty in studying corrosion in this setting is the limited access for sensors in the small, confined spaces inside of pipes. Efird [5] proposed experimentally creating plots of the film breakdown shear stress vs. velocity for given alloys. He then argued this data could be applied to other settings where similar conditions were encountered. To avoid measurements inside of pipes, several external geometries have been proposed (e.g. rotating disk, rotating hemisphere) in order to determine the effect of fluid velocity on corrosion rates. The most success was by Silverman [6], who proposed matching the wall shear stress between pipe flows and a rotating cylinder electrode (RCE). As long as the flow regimes were the same (laminar or turbulent), he argued that the mechanism affecting corrosion due to fluid velocity would be captured. Silverman also cautions this line of reasoning is merely a hypothesis.

Despite Silverman’s unproven hypothesis, his method became the standard approach in capturing the impact of fluid dynamics on corrosion rates in pipes. Some years later, Dierich et al. [7] experimentally measured the turbulent velocity boundary layer very near a spinning cylinder in a quiescent fluid. They found that the law of the wall theory developed based on flat plate flow was still valid despite curved streamlines near a curved surface. Hwang et al. [8,9] completed direct numerical simulations (DNS) of flow about a rotating cylinder at low Reynolds numbers. They also found that the laminar sublayer, buffer layer, and logarithmic outer region existed in the boundary layer much like any other geometry. Since corrosion reactions occur at the surface, Hwang et al. [9] argue that current corrosion models that assume a linear velocity profile are valid since this zone is within the laminar sublayer of the boundary layer. Arguments such as these tend to support Silverman’s approach.

However, corrosion models typically underperform in their predictive ability so clearly there are some missing puzzle pieces. For example, Paolinelli and Carr [10] show that corrosion films on very fast spinning cylinders may fail due to inertial forces rather than flow rates. Side effects such as these are unavoidable when attempting to use a spinning cylinder to represent pipe flow. If not taken into account, side effects such as these can negatively impact the predictive ability of the associated models.

The goal of the present work is to probe more deeply the pipe flow/spinning cylinder analogy. In what follows, the governing hydrodynamic equations are presented along with analytical solutions for laminar cases. These are compared with numerical simulations, and the numerical simulations also extend into the turbulent regime. A correlation between the angular velocity of a spinning cylinder and the average velocity inside of a pipe that produces equal wall shear stresses is presented. This relation can aid in the experimental setup to investigate the shear stress analogy. Effects of the outer boundary placement in spinning cylinder tests are also discussed.

GOVERNING EQUATIONS
For incompressible fluid flow, the mass and momentum equations must be solved in order to determine the pressure and velocity fields. The Navier-Stokes equations are written

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0
\]  
(1)

\[
\frac{\partial (\rho \mathbf{u})}{\partial t} + \mathbf{u} \cdot \nabla (\rho \mathbf{u}) = -\nabla p + \nabla \cdot [\nu \nabla (\rho \mathbf{u})] + \mathbf{F}
\]  
(2)

where \(\rho\) is density, \(t\) is time, \(\mathbf{u}\) is the velocity vector, \(p\) is pressure, \(\nu\) is the kinematic viscosity, and \(\mathbf{F}\) is the body force term, which includes body forces due to the electric field. This is where the electric potential in an electrochemical flow couples to the hydrodynamics, and it will be an important part of future corrosion studies. For incompressible flows, the energy equation is not necessary to close the system of equations since density may be considered constant. For turbulent simulations, additional transport equations are required, and the \(k-\omega\) SST (shear stress transport) [11,12] turbulence model will be used in this work. This two equation model describes the transport of turbulent kinetic energy, \(k\), and dissipation rate, \(\omega\).
In corrosion modeling, the foregoing hydrodynamic equations are coupled with the Nernst-Planck equation through the velocity vector. Wang and Kang [13] give a review of the Nernst-Planck equation development. For the \(i\)th ion species in the solute, the mass conservation equation describing transport and reaction can be written in the general form [14]:

\[
\frac{\partial C_i}{\partial t} + \nabla \cdot \mathbf{J}_i + \lambda_i C_i = R_i
\]

(3)

where \(C_i\) denotes the ionic concentration, \(\mathbf{J}_i\) is the species flux, \(\lambda_i\) is a radioactive decay constant, and \(R_i\) is the rate at which the \(i\)th species is produced or consumed by chemical reactions. The flux term contains advection, diffusion, dispersion, and electrochemical migration. Neglecting dispersion, the flux term is

\[
\mathbf{J}_i = -\frac{e z_i D_i}{kT} C_i \nabla \Psi - D_i (\nabla C_i + C_i \nabla \ln \gamma_i) + C_i \mathbf{u}
\]

(4)

The first term on the right is the electrochemical migration term, the second is due to aqueous diffusion, and the third term is the advective transport. Here, \(z_i\) is the ion algebraic valence, \(D_i\) the diffusivity, \(\gamma_i\) the activity coefficient of the \(i\)th species, \(e\) the absolute charge of the electron, \(k\) the Boltzmann constant, and \(T\) the absolute temperature. The quantity \(\Psi\) is the local electrical potential caused by the ionic distribution. Expanding the divergence of \(\mathbf{J}_i\) using the rules of vector calculus gives

\[
\nabla \cdot \mathbf{J}_i = -\nabla \cdot \left( \frac{e z_i D_i}{kT} C_i \nabla \Psi \right) - \nabla \cdot (D_i \nabla C_i) - \nabla \cdot (D_i C_i \nabla \ln \gamma_i) + \nabla \cdot (C_i \mathbf{u})
\]

(5)

If there is no polarization, radiation, or chemical reactions, the familiar form of the Nernst-Planck equation is recovered

\[
\frac{\partial C_i}{\partial t} + \mathbf{u} \cdot \nabla C_i = \frac{e z_i D_i}{kT} \nabla \cdot (C_i \nabla \Psi)
\]

(7)

The electrical potential due to the ionic distribution is governed by the Poisson equation

\[
\nabla \cdot (\varepsilon_r \varepsilon_0 \nabla \Psi) = -\rho_E = -\sum_i e z_i C_i
\]

(8)

where \(\varepsilon_r\) is the local fluid dielectric constant and \(\varepsilon_0\) is the permittivity of a vacuum. If these are both constant, the electrical potential can be expressed

\[
\nabla^2 \Psi = \frac{-\rho_E}{\varepsilon_r \varepsilon_0} = -\sum_i e z_i C_i
\]

(9)

The ion concentrations can be used in the Butler-Vollmer equation to determine current density. The details are beyond the scope of this paper, but this eventually leads to the mass loss due to corrosion. However, mass loss from surfaces will alter the surface roughness (and thus, friction factor), which will impact the hydrodynamic solution. A relationship between the mass loss, surface area, and friction factor does not exist, and the work in this paper is intended as an initial step in developing this relationship.
EXACT SOLUTIONS

The Navier-Stokes equations can be solved for steady, laminar, incompressible flow about a spinning cylinder and inside of a straight pipe. The pipe flow problem in the fully developed region is a classic fluids case called Poiseuille flow \[15\]. Using the cylindrical coordinate system from Figure 1a, the momentum equation in the \(z\)-direction reduces to

\[
\frac{1}{r} \frac{d}{dr} \left( r \frac{dv_z}{dr} \right) = \frac{1}{\mu} \frac{\partial p}{\partial z} \tag{10}
\]

where \(\mu\) is the dynamic viscosity. For pipe flow, the following non-dimensionalization is used:

\[
r^* = \frac{r}{R} ; \quad z^* = \frac{z}{R} ; \quad v_z^* = \frac{v_z}{V} ; \quad p^* = \frac{p}{\mu V/R} \tag{11}
\]

where \(V\) is the average pipe velocity (to be defined later) and \(R\) is the radius of the pipe. Equation 10 is now recast as

\[
\frac{1}{r^*} \frac{d}{dr^*} \left( r^* \frac{dv_z^*}{dr^*} \right) = \frac{\partial p^*}{\partial z^*} \tag{12}
\]

Applying the no slip condition at the wall \(v_z^*(1) = 0\) and forcing the velocity at the centerline to be finite, the solution becomes

\[
v_z^*(r^*) = \frac{1}{4} \left( \frac{\partial p^*}{\partial z^*} \right) \left( 1 - r^{*2} \right) \tag{13}
\]

where the negative sign from the pressure drop in the \(z\)-direction term has been included.

The average velocity is the volumetric flow rate divided by the cross-sectional area of the pipe. Since the average velocity is used to non-dimensionalize the axial pipe velocity, it should be determined now in a dimensional form. If the velocity in Eq. 13 is put back in dimensional form, integrated from 0 to \(R\), and divided by the cross-sectional area \(\pi R^2\), the average velocity becomes

\[
V = \frac{\Delta p D^2}{32 \mu L} \tag{14}
\]

where \(\Delta p\) is the pressure drop over some length of pipe \(L\) and \(D\) is the pipe diameter. The interested reader can follow the details in Gerhart et al. \[15\], but this enables the shear stress on the pipe wall to be expressed compactly as

\[
\tau_{pipe} = \frac{8 \mu V}{D} \tag{15}
\]

For a 2D cylinder spinning in a quiescent fluid, (see Figure 1b), an analytical solution exists assuming steady, laminar, incompressible, and axisymmetric flow in the absence of body forces with no velocity component in the radial direction. In this case, the momentum equation in the \(\theta\)-direction reduces to

\[
\frac{1}{r} \frac{d}{dr} \left( r \frac{dv_\theta}{dr} \right) = 0 \tag{16}
\]

For cylindrical flow, the following non-dimensionalization is used:

\[
r^* = \frac{r}{R} ; \quad v_\theta^* = \frac{v_\theta}{\omega R} \tag{17}
\]
where \( R \) is the radius of the cylinder and \( \omega \) is the angular velocity. In non-dimensional form, Eq. 16 becomes

\[
\frac{1}{r^*} \frac{d}{dr^*} \left( r^* \frac{dv^*_\theta}{dr^*} \right) - \frac{v^*_\theta}{r^*} = 0
\]

(18)

Applying boundary conditions that the velocity at the surface of the cylinder must be \( v^*_\theta(1) = 1 \) and that the velocity must be bounded as the radial position goes to infinity \( (v^*_\theta(r^* \to \infty) = \text{bounded}) \), the resulting solution for velocity is given by

\[
v^*_\theta(r^*) = \frac{1}{r^*}
\]

(19)

The velocity profile can be inserted into the shear stress tensor in cylindrical coordinates to obtain the shear stress on the surface of the cylinder. Since this will be compared with the dimensional form of the pipe shear stress later on, we use a dimensional form again here:

\[
\tau_{\text{cyl}} = \mu \left[ r \frac{\partial}{\partial r} \left( \frac{v^*_\theta}{r} \right) + \frac{1}{r} \frac{\partial v^*_r}{\partial \theta} \right]_{r=R} = -2\mu \omega
\]

(20)

Note that the second term in brackets is zero since it has been assumed that the flow has no radial component.

If the domain is limited in the far field, as it would be in any experiment, the boundary condition away from the cylinder would also be a no slip condition at those far field walls, or \( v^*_\theta(n) = 0 \), where \( n \) is the number of cylinder radii to the outer wall. Now the velocity profile is written

\[
v^*_\theta = \frac{1}{n^3 - 1} \left( -r^{n^2} \right) + \frac{n^3}{r^*}
\]

(21)

The shear stress on the surface of the cylinder can be found the same way as before. Using L'Hospital's Rule, it can be shown that the shear stress on the cylinder is equal to that in Eq. 20 in the limit where \( n \) approaches infinity.

Figure 1: Coordinate systems for a) pipe and b) spinning cylinder flow.

NUMERICAL APPROACH

All numerical simulations were completed using the open source software package OpenFOAM\(^1\) [16]. This software is a collection of solvers and utilities designed to solve partial differential equations related to fluid dynamics. The native simpleFoam solver was used in all simulations. This solver is designed for use in steady, incompressible cases with or without turbulence modeling. As the name implies, this solver uses the SIMPLE (semi-implicit method for pressure-linked equations) algorithm to iteratively solve the coupled pressure and velocity equations [17].

On solid boundaries, the no slip condition is enforced \( (u = 0) \), and the pressure gradient normal to the surface is set to zero. For spinning cylinder flows, the velocity in the far field is set to a zero gradient, and the pressure is set to the

\(^1\) OpenFoam is an open source software package available from https://openfoam.org.
ambient value. In pipe flows, the inlet velocity is prescribed, and the outlet velocity is a zero gradient boundary. The pressure at the inlet is treated as a zero gradient boundary, and the outlet is set to the ambient value. For turbulent flows, the turbulence variables are set as directed by Menter [11].

**CFD VALIDATION**

*Pipe Flow*

For pipe flows, the flow is considered laminar for Reynolds numbers less than about 2100. The Reynolds number is defined based on the internal diameter as

\[
\text{Re}_D = \frac{VD}{v}
\]

To ensure fully laminar flow, simulations were run at \(\text{Re}_D = 100\). If the solution is assumed to be axisymmetric, the domain only needs to consist of a portion of the pipe cross-section that runs the length of the pipe. A 90-degree section as shown in Figure 2 was chosen for the domain over a single 5-degree wedge since this would more readily extend to include 3D effects if necessary in the future. The coarse grid in Figure 2 contains 20 cells in the radial direction, 10 cells in the circumferential direction, and 50 cells along the length of the pipe. This was considered the coarse configuration. A fine grid was also created with double resolution in all three directions. The length of the pipe extends approximately 15 radii in the z-direction.

![Figure 2: Coarse grid depicting quarter domain.](image)

The entrance length defines the region at the inlet of the pipe where the velocity profile has not fully developed, meaning that it is not constant with respect to axial direction. For laminar flow, this length is usually taken as [15]

\[
\frac{L}{D} = 0.06 \text{Re}
\]

For the current flow and geometry, the fully developed region starts near 76% along the length of the pipe. Figure 3 shows contours of velocity magnitude along the entire length of the pipe. Near the end of the pipe, constant contours indicative of the fully developed region are evident. The growing boundary layer along the pipe wall beginning at the inlet is also clearly visible. The velocity profile from a cross-section in the fully developed region is shown in Figure 4. There is excellent agreement between the two simulations and theory here. The centerline velocity is twice the average velocity as expected. In Figure 5, the shear stress along the length of the wall is plotted. For the theoretical data, only the expected value for the fully developed region is used. Both simulations asymptote towards the theoretical value. There is disagreement between the coarse and fine meshes near the pipe inlet in terms of wall shear stress. Although it will not be pursued here since it does not impact the fully developed region, a finer mesh would be necessary to better resolve this region if desired.
Figure 3: Velocity contours along length of pipe from fine grid simulation.

Figure 4: Velocity profile in the fully developed region of the pipe (taken at $z/L = 0.9$).

Figure 5: Wall stress along length of pipe. The theoretical value is only meant to indicate the value once the flow is fully developed.

Spinning Cylinder Flow
For a rotating cylinder, the limit for laminar flow corresponds to a rotational Reynolds number between 40 and 60 [18]. The rotational Reynolds number is defined
To ensure fully laminar flow, simulations were run at $Re_r = 9.62$. A close up of the coarse grid is shown in Figure 6. Once again, due to symmetry, only a quarter of the domain is modeled. The grid contained 40 cells in the circumferential direction, 100 cells in the wall normal direction, and extended over 130 cylinder radii away from the surface. A fine grid with double resolution in both directions was also used.

![Figure 6: Close-up view of grid near cylinder surface.](image)

Based on the analytical solution, which assumes no radial velocity component, the velocity contours should appear as concentric circles. Velocity contours are shown for the fine grid in Figure 7, and indeed, the flow exhibits no radial component. Figure 8 shows the velocity distribution from the cylinder wall outward for the analytical solution and for the two grids. The simulations exhibit grid independence and excellent agreement with theory. Finally, the shear stress at the wall predicted by the simulations was within 0.08% of the theoretical value given by Eq. 13.

![Figure 7: Velocity contours near cylinder surface. The concentric circles are indicative of the absence of a radial component in the velocity.](image)
RELATIONSHIP BETWEEN SHEAR STRESSES

In order to match the wall shear stress between a rotating cylinder and fully developed pipe flow, analytical solutions for laminar flows and simulations for turbulent flows can be utilized. Figures 9 and 10 show the accumulated data for the rotating cylinder and pipe, respectively. The equations shown are in terms of the non-dimensional variables on the axes. In Figure 9, for example, instead of $x$ in the equation shown, it is really $Re$. Not surprisingly, the pipe flow data closely matches that of a Moody diagram [15] since wall shear stress is directly correlated to friction factor. Data from Silverman [6] and a RCE manufacturer [19] are included in the plots for reference.

**Figure 9:** Predicted shear stress on a spinning cylinder including data from Silverman [6] and Pine Research [19].
Given the discontinuity in data between laminar and turbulent pipe flow, separate correlations for the two regimes is a better approach. The variables must be put back in dimensional form before matching shear stress magnitudes. Rather than using the laminar correlations from Figs. 9 and 10, the analytical solutions can be used directly. Matching shear stress magnitudes from Eqs. 15 and 20 gives

\[ \tau_{cyl} = \tau_{pipe} \quad (25) \]

\[ 2\mu\omega = \frac{8\mu U_\infty}{D} \quad (26) \]

Assuming the same viscosity for the rotating cylinder and pipe flow, the final functional relationship between the two geometries to match shear stress is

\[ \omega = \frac{4V}{D} \quad \text{OR} \quad V = \frac{\omega D}{4} \quad (27) \]

For clarity, in these equations, \( \omega \) is the angular velocity of the spinning cylinder in rad/s, \( V \) is the average velocity of the pipe flow in m/s, and \( D \) is the diameter of the pipe in m. This relationship is valid for \( Re_r < 40 \) and \( Re_D < 2100 \). Equation 27 enables ease in setting up the analogous experiments where shear stresses need to be matched.

There is no theoretical or readily available empirical correlation for spinning cylinder turbulent flow. Using the relation in Figure 9, the shear stress in this region is

\[ \tau_{cyl} = \frac{1}{2}\rho \omega^2 R^2 (Re_r^{0.1315}) \log Re_r Re_{1.3164}^{10^{0.8812}} \quad (28) \]

Due to the scale of the data, a fit any less complicated does not adequately describe the data. For turbulent pipe flow, a correlation directly from the Moody chart is used since it is already based on experiment and it has been in use for decades. The friction factor \( f \) in the Moody diagram is famously given by the Colebrook formula

\[ \frac{1}{\sqrt{f}} = -2.0 \log \left( \frac{\epsilon/D}{3.7} + \frac{2.51}{Re_D \sqrt{f}} \right) \quad (29) \]

where \( \epsilon \) is the roughness of the pipe. The implicit nature for friction factor in this formula makes it difficult to use so one popular approximation to this formula is the Haaland equation, which is
For a smooth pipe, $\varepsilon$ is zero, and this term falls out of the above correlations. Wall shear stress in a pipe can be related to the friction factor \cite{15}, which gives the following formula for shear stress

$$
\frac{1}{\sqrt{f}} = -1.8 \log \left[ \left( \frac{\varepsilon}{D} \right)^{1.11} + \frac{6.9}{\text{Re}_D} \right]
$$

(30)

Matching the two equations for turbulent shear stress (Eqs. 28 and 31), only the density cancels, and we are left with

$$
\frac{1}{4} \rho V^2 = \frac{1}{8 \left[ -1.8 \log(6.9/\text{Re}_D) \right]^2}
$$

(31)

Finding an analytic or asymptotic functional relationship between $\omega$ and $V$ is not easy, if even possible. However, the above equation can be solved iteratively for varying values of $\omega$, $V$, $R$, $D$, and $\nu$, and a curve or set of curves can be fit to the resulting values. These primitive variables also scale better than non-dimensional shear stress and Reynolds number, which means that polynomials will be better able to describe the fit.

Since $R$ is 0.0075 m for the RCE that will be used in experiments, this value was held constant. The pipe diameter could vary, but for now it is also kept constant at 0.0127 m. The viscosity was given potential seawater values ranging from $8.0 \times 10^{-7}$ to $1.5 \times 10^{-6}$ m$^2$/s. Solving the above equation iteratively over a range of angular velocities for which the flow is turbulent based on Reynolds number, the relationship between angular velocity and average pipe velocity emerges as shown in Figure 11. The viscosity has a negligible impact on the solution (although values outside of the range shown here did have an impact), and, surprisingly, the relationship between angular velocity and average pipe velocity appears linear. Fitting a line, the functional relationship is given by

$$
\omega = 154.18V - 1.7733 \quad \text{OR} \quad V = 0.0065\omega + 0.0116
$$

(33)

![Figure 11: Relationship between average pipe velocity and RCE angular velocity for varying viscosities.](image)

To test the developed correlations, simulations were run for both laminar and turbulent cases. The runs are summarized in Table 1. The magnitude of the shear stress on the wall is given for the cylinder and pipe, and the final column gives the difference between the two defined as
\[ \Delta \tau = \left| \frac{\tau_{\text{cyt}} - \tau_{\text{pipe}}}{\tau_{\text{cyt}}} \right| \times 100\% \]  

All but the final value at \( \omega = 400 \text{ rad/s} \) are within 6%. One source of error is that the pipe correlation was determined from the Moody diagram and not CFD. As the Reynolds number increased, different CFD grids with differing near wall spacings were needed to converge the solutions. This should be further explored to potentially reduce some of the disagreement in Table 1.
**Table 1**: Predicted Values of Shear Stress for Cylinder and Analogous Pipe Flows

<table>
<thead>
<tr>
<th></th>
<th>Cylinder</th>
<th>Pipe</th>
<th>∆τ</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ω, 1/s</td>
<td>Re&lt;sub&gt;r&lt;/sub&gt;</td>
<td>R, m</td>
<td>ν, m&lt;sup&gt;2&lt;/sup&gt;/s</td>
</tr>
<tr>
<td>Lamina</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.208</td>
<td>10</td>
<td>0.0075</td>
<td>1.17E-06</td>
</tr>
<tr>
<td>0.416</td>
<td>20</td>
<td>0.0075</td>
<td>1.17E-06</td>
</tr>
<tr>
<td>0.747</td>
<td>30</td>
<td>0.0075</td>
<td>1.40E-06</td>
</tr>
<tr>
<td>Turbulent</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>4020</td>
<td>0.0075</td>
<td>1.40E-06</td>
</tr>
<tr>
<td>80</td>
<td>4500</td>
<td>0.0075</td>
<td>1.00E-06</td>
</tr>
<tr>
<td>150</td>
<td>9480</td>
<td>0.0075</td>
<td>8.90E-07</td>
</tr>
<tr>
<td>400</td>
<td>19200</td>
<td>0.0075</td>
<td>1.17E-06</td>
</tr>
</tbody>
</table>

**BOUNDING DOMAIN EFFECTS**

Experimentalists need to know how close is too close for the outer wall of their container when planning an RAE experiment. Plotting a few solutions to Eq. 21, the trend can be observed in Figure 12. For any reasonable number of radii outward, the near field velocity profile is not impacted so derived quantities such as shear stress should not be impacted either. Indeed, by 20 or 40 radii, the impact appears to be negligible on near field phenomena.

Assuming shear stress is the dominant factor in the corrosion analogy, the impact of the confined test domain on the observed wall shear stress should be examined. Using Eq. 35, the percent difference between the wall shear stress on a spinning cylinder in an infinite domain and a confined domain is plotted in Figure 13. Before less than five cylinder radii away from the spinning cylinder, the difference is less than 5%. By ten radii, the difference is almost negligible. Small confined test spaces do not appear then to be a problem if wall shear stress is indeed the only dominant factor from a hydrodynamics standpoint. It must also be emphasized here that the effects on wall distance presented here are only for laminar flows. Turbulent boundary layers are larger so these results are qualitative at best for turbulent flows.

![Figure 12: Analytical velocity profiles for wall bounded spinning cylinder flow.](image)

\[ \Delta \tau_{cyl} = \left| \frac{\tau_{cyl} - \tau_n}{\tau_{cyl}} \right| \times 100\% \]
Figure 13: Percent difference in theoretical shear stress on a spinning cylinder between an infinite and bounded domain as defined in Eq. 35.

SUMMARY
Analytical solutions and numerical simulations of flows through straight sections of pipe and about a spinning cylinder were used to create functional relationships between the average pipe velocity and angular velocity of the cylinder. The matching was based on equal wall shear stresses between the two flows. Despite the complicated hydrodynamics equations, a linear relationship was found for both laminar and turbulent flows. This relationship will aid in setting up corrosion experiments where the shear stress is to be matched. The extent of the outer boundary in rotating cylinder experiments was also shown to be insignificant if it is placed at least 10 radii out from the cylinder in laminar flows.

ACKNOWLEDGEMENTS
The authors would like to thank the Cross Platform System Development Program under Ms. Danesha Gross and the Office of Naval Research Summer Faculty Program administered by Dr. Jack Price for their aid in this work. Any opinions, findings, conclusions or recommendations expressed in this document are those of the authors and do not necessarily reflect the views of the NAVSEA or Naval Surface Warfare Center, Carderock Division. This document has been approved as Distribution A: Approved for public release; distribution unlimited.

REFERENCES


